NUS-JSPS Workshop on Algebra
Singapore-Warwick Workshop in Geometry & Topology

ABSTRACTS

Monday, 9 July 2001

Compactification of orbits of hyperbolic elements in semisimple symmetric pairs

Jiro Sekiguchi
Tokyo University of Agriculture and Technology

Let $\mathbf{g}$ be a semisimple Lie algebra and let σ be an involution. Then we obtain a direct sum decomposition $\mathbf{g} = \mathbf{h} + \mathbf{q}$ where $\mathbf{h}$ and $\mathbf{q}$ are the +1 and -1 eigenspaces of σ, respectively. Let G be the group of inner automorphisms of $\mathbf{g}$.

An element $x \in \mathbf{q}$ is said to be hyperbolic if x is semisimple and all the eigenvalues of $Ad(x)$ are real. Let $\mathbf{p}(x)$ be the direct sum of eigenspaces of $Ad(x)$ with non-negative eigenvalues. Then $\mathbf{p}(x)$ is a parabolic subalgebra of $\mathbf{g}$.

Let $P(x)$ be the parabolic subgroup of G with Lie algebra $\mathbf{p}(x)$. If H is the identity component of the fixed point subgroup G^σ of G with respect to σ, then $H \cdot x$ is naturally embedded into the flag variety $G/P(x)$.

In my talk, I will study the H-orbital structure of $G/P(x)$ in detail when the eigenvalues of $Ad(x)$ are $0, \pm 1$.

Acyclic groups, large and small

Jon Berrick
National University of Singapore

Acyclic groups form a remarkable, yet little-studied, class. The "large" examples occur throughout mathematics, as automorphism groups of mathematical structures. The "small" examples occur more sporadically, yet even here patterns exist. In this survey, with an emphasis on the history of the subject, I shall try to indicate where acyclic groups may be found, and highlight their importance, their properties and some interesting patterns.

ABSTRACTS

On Gorenstein log del Pezzo surfaces

Qiang Ye
National University of Singapore

We first present the complete list of the singularity types of the Picard number one Gorenstein log del Pezzo surfaces and the number of the isomorphism classes for the given singularity type. Then we give a method to find out all singularity types of Gorenstein log del Pezzo surfaces. As an application, we present the complete list of the singularity types of the Picard number two Gorenstein log del Pezzo surfaces. Finally we give a complete classification of the Relatively minimal Gorenstein log del Pezzo surfaces and the number of the isomorphism classes with the given singularity type.

Moduli spaces and the minimal model program

Brendan Hassett
Chinese University of Hong Kong

We consider weighted pairs, consisting of a projective variety and an effective divisor with rational coefficients. For example, take a configuration of points on the projective line, and assign to each a nonnegative number no greater than one. Using the log minimal model program to describe the limiting degenerate pairs, one can construct compact moduli spaces of weighted stable pairs. The limits that arise depend on the choice of the weights, and varying the weights yields a collection of moduli spaces, often related by explicit birational modifications. These modifications themselves often admit elegant interpretations in terms of the log minimal model program, as applied to the moduli spaces. We compare moduli spaces of weighted pairs to compactifications of configurations of linear subspaces arising from Geometric Invariant Theory, and to compactifications of moduli of pointed rational curves studied by Kapranov, Losev, and Manin. Finally, we will state some results on higher dimensional examples like plane curves.

ABSTRACTS

Singular unitary representations and theta correspondence

Eng-Chye Tan
National University of Singapore

This is an expository lecture to introduce the use of duality in the problem of understanding the unitary dual for classical groups. We will explain the notion of duality and the rank of a representation; both introduced by Roger Howe. Then we will highlight how J.S. Li's results on the duality correspondence in the stable range as well as the lifting of discrete series representations give rise to a qualitative description of part of the unitary dual in some cases. Examples will be given using $(O(p,q),SL(2,\mathbf{R}))$ and $(Sp(p,q), O^*(2n))$.

Tuesday, 10 July 2001

Logarithmic transformation on elliptic surfaces

Noboru Nakayama
RIMS, Kyoto

The notion of logarithmic transformation of elliptic surfaces was discovered by Kodaira in his study of multiple fibers. This is a surgery along fibers. The transform from a projective surface need not to be projective. I will explain a cohomological interpretation of the logarithmic transformation and a condition for the transform to be projective.

Families of Fourier-Mukai transforms in mirror symmetry

Balazs Szendroi
University of Warwick

I explain how Kontsevich' homological mirror symetry conjecture implies the existence of families of equivalences of derived categories over (marked) moduli spaces of Calabi--Yau varieties. I will discuss in detail the case of K3 surfaces and Calabi--Yau threefolds. In the latter case, the families of equivalences obtained satify rather intricate relations.

Finite automorphism groups on rational normal surfaces

De-Qi Zhang
National University of Singapore

We classify minimal pairs (X, G) for smooth rational projective surface X and finite group G of automorphisms on X. Mori's extremal ray theory and recent results of Alexeev and also Ambro on the existence of good anti-canonical divisors are used. In the last part, we also consider pairs where X may have some Du Val singularities.

ABSTRACTS

Automorphism groups in a family of K3 surfaces

Keiji Oguiso
University of Tokyo

A few facts concerning the phrase "the automorphism groups become larger at special points of the moduli of K3 surfaces" are presented. It is also shown that the automorphism groups are of infinite order over a dense subset in any one-dimensional non-trivial family of projective K3 surfaces.

Singular potentials on Hermitian symmetric spaces

Wing-Keung To
National University of Singapore

In this talk, we describe the construction of certain singular potential functions on Hermitian symmetric spaces of non-compact type, and discuss their applications to bounding Seshadri constants of the canonical bundles on their compact quotients and bounding volumes of their complex analytic subvarieties.

ABSTRACTS

Wednesday, 11 July 2001

Some connections between coding theory and modular forms

Michio Ozeki
Yamagata University

In the present talk we discuss various connections between the ideas in coding theory and their applications to the theory of Siegel modular forms and Jacobi forms. More precisely we talk on:

(1) construction of modular forms by means of weight enumerators and related polynomials,
(2) construction of Siegel modular forms by means of multiple weight enumerators of codes,
(3) construction of Jacobi forms by means of Jacobi polynomials for codes,
(4) construction of Hilbert modular forms by means of Lee weight enumerators of codes over finite fields,
(5) comparing two kinds of Mass formulas in the coding theory and in the lattice theory, and
(6) applications of ideas in coding theory to some arithmetical problems.

On the graded rings coming from coding theory

Manabu Oura
Sapporo Medical University

The relations among coding theory, invariant theory, and the theory of modular forms are known and have been studied by several mathematicians. In my talk, I describe some results on the graded rings coming from coding theory, one of which has a close connection to the graded ring of Siegel modular forms in genus 4. Results I will talk are obtained jointly with Professor Freitag, and with Professors Dougherty and Gulliver.

ABSTRACTS

Curves over finite fields with many rational points

Chaoping Xing
National University of Singapore

Starting from the zeta function of a smooth, irreducible,projective curve over finite fields, we introduce the Weil theorem and the Hasse-Weil bound. Serre's bound is also established based on the zeta function of a curve. Besides, maximal and optimal curves are defined.

Based on the correspondence between curves and global function fields over finite fields, we study two constructions of curves over finite fields with many rational points using class field theory. Precisely speaking, we consider the Hilbert class fields and narrow ray class fields controlled by conductors.

(DEPARTMENTAL COLLOQUIUM TALK)
On a theorem of Jordan

J P Serre
College de France

Abstract not available.

ABSTRACTS

Thursday, 12 July 2001

Semistable sheaves in mixed characteristic

Adrian Langer
University of Warwick

I would like to talk about my proof of Maruyama's conjecture on the boundedness of slope semistable sheaves on a projective variety defined over a noetherian ring. My approach also gives a new proof of the boundedness for varieties defined over a characteristic zero field. This result implies that in mixed characteristic the moduli spaces of Gieseker semistable sheaves are projective schemes of finite type. The proof uses a new inequality bounding slopes of the restriction of a sheaf to a hypersurface in terms of its slope and the discriminant. This inequality also leads to effective restriction theorems in all characteristics, Improving earlier results in characteristic zero. At the end of my talk I will probably mention some applications to sheaves with operators and Higgs bundles.

A generalization of conjectures of Bogomolov and Lang over finitely generated fields

Atsushi Moriwaki
Kyoto University

In my talk, I will explain a generalization of conjectures of Bogomolov and Lang in terms of an arithmetic Neron-Tate height pairing over a finitely generated field.

Using algebraic curves to construct good linear codes

San Ling
National University of Singapore

Constructing codes with good parameters is an important problem in coding theory. Various tools from algebra, number theory, geometry, combinatorics, etc., have been employed for the construction of linear codes. It is generally believed that constructing new linear codes, especially binary ones of small lengths, is becoming more and more difficult, given that much research has been done in this area.

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Algebraic geometry has been extensively used in the construction of codes since the discovery of Goppa's geometry codes. In order to get good Goppa's geometry codes, one has to find algebraic curves with as many rational points as possible. However, it seems impossible to obtain many good q-ary linear codes by directly applying Goppa's construction for small q since curves over small finite fields have few rational points compared with their genera.

In this talk, we report on some recent constructions of linear codes using closed points of higher degrees on algebraic curves. Many new linear codes, binary as well as non-binary ones, have been obtained from these methods.

Two characterizations of symmetric domains by the Laplace-Beltrami operator and their geometric background

Takaaki Nomura
Kyoto University

As the title indicates, we give two characterizations of a homogeneous Siegel domain being symmetric in terms of the Laplace-Beltrami operator, and we will also explain their geometri background.

Group determinant and the Capelli Identity

Toru Umeda
Kyoto University

The classical Capelli identity is, on the one hand, a formal multiplication formula of determinants for the matrices with non-commutative entries (differential operators). On the other hand, from the representation-theoretic side, it is an equality between the two invariant differential operators, and is closely related to the so-called multiplicity-free actions.

In my talk, I will show that a Capelli type multiplication formula of group determinants holds. This is also related to some (most trivial) decomposable multiplicity-free actions.

Friday, 13 July 2001

Degenerate principal series representations and Gelfand-Zetlin bases

Hung-Yean Loke
National University of Singapore

A degenerate principal series representation (DPSR) is defined as a principal series representation induced from a finite dimensional representation of a parabolic subgroup. It is an interesting problem to find all its irreducible subquotients.

First I will give a brief description of the method used in the investigation of K-multiplicity free DPSR. These are due to R. Howe-E. C. Tan, Klimyk et al, S. T. Lee, K. Johnson, S. Sahi and G. Zhang. In particular, Klimyk et al has a series of papers studying such problems using the Gelfand-Zetlin bases of finite dimensional representations of U(n) and Spin(n).

Next I will report on my joint work with Soo Teck Lee on the Jordan-Holder series of families of DPSRs of U(p,q), $Spin_0(p,q)$ and Sp(p,q) which are not K-multiplicity free. We do this by embedding these DPRS into K-multiplicity free DPSR and we study their reducibility and unitarity using Gelfand Zetlin bases.

Finally I describe how to extend the results on $SO_0(p,q)$ to O(p,q).

The database of K3 surfaces in Magma

Gavin Brown
University of Sydney

I will report on a database of K3 surfaces in the computer algebra system Magma. This is built using Hilbert series methods which describe the degrees of variables and equations in some graded ring. The methods generate candidates for K3 surfaces which one must then show actually exist with particular properties. An interesting aspect is the way in which the inductive construction of the database does some of the work required to prove the existence of the objects which it contains.

I will show examples of K3 surfaces and give a description of the Hilbert series methods used to find candidates. Then I will indicate how the method of "unprojection" in conjunction with the building of the database can show the existence of these rather complicated graded rings.

This is joint work with Selma Altinok and Miles Reid. It could be viewed as one piece in a long series of Warwick PhD theses.

Cheltsov rigidity of Fano 3-folds

Daniel Ryder
University of Warwick

The aim of the talk will be to present examples of the calculations used to describe K=0 fibrations birational to (birationally rigid) Fano varieties. In general terms, the methods used are extensions of those employed in proofs of birational rigidity (e.g. in [Corti, Pukhlikov, Reid]). I will outline one or two results of Ivan Cheltsov and also describe specifically some new calculations on Fano hypersurfaces in weighted projective spaces.

Braids, mapping class groups and the homotopy groups

Jie Wu
National University of Singapore

This is a joint work with Jon Berrick, Fred Cohen and Yan Loi Wong.

Let M be a manifold. A braid of n strings over M is called Brunnian if it becomes a trivial braid when we remove any one of the strings. For instance, the well-known Borromean ring is a link by closing up a Brunnian braid of 3-strings over the disc. Let $Br_n(M)$ be the group of Brunnian braids of n strings over M. The inclusion D^2 into S^2 induces a group homomorphism $Br_n(D^2) \to Br_n(S^2)$. In this talk, we show that there is a exact sequence

$$1 \to Br_{\{n+1\}}(S^2) \to Br_n(D^2) \to Br_n(S^2) \to \pi_{\{n-1\}}(S^3) \to 1$$

for $n \geq 5$. This gives a connection between the Brunnian braids and the homotopy groups.

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K3 surfaces, Fano varieties and graded rings

Miles Reid
University of Warwick

Classical Fano 3-folds are studied in terms of their K3 surface and canonical curve sections. Mori theory treats Fano 3-folds with terminal singularities, typically quotient singularities, and in good cases, the section is a K3 surface with Du Val singularities, polarised by a Weil divisor, and the varieties are studied as polarised varieties, in terms of their graded rings. As well as providing the relevant class of varieties for the 3-fold minimal model program, this study leads to a huge number of new examples of varieties and graded rings.

For a colloquial introduction to these ideas, see the preface to our book

A. Corti and M. Reid (editors.), Explicit birational geometry of 3-folds, CUP/LMS lecture notes 2000
http://www.maths.warwick.ac.uk/~miles/Unpub/CPR_book/preface.ps