Recent Developments in Algebraic Geometry

24 January 2018 (Wednesday)
10.00am to 5.30pm
S17-05-12 (Seminar Room 4)

Organisers:
Zhang De-Qi (NUS)
JongHae Keum (KIAS)
Recent Developments in Algebraic Geometry
24 January 2018

Programme

10.00am – 11.00am  A surface with discrete and non-finitely generated automorphism group
                      Tien-Cuong Dinh
                      National University of Singapore

11.15am – 12.15pm  A Lefschetz fibration structure on minimal symplectic fillings of a quotient surface singularity
                      Jongil Park
                      Seoul National University

14.00pm – 15.00pm  Threefold in $\mathbb{P}^5$: multisecant and regularity
                      Sijong Kwak
                      Korea Advanced Institute of Science and Technology (KAIST)

15.15pm – 16.15pm  On polarized and amplified endomorphisms of normal projective varieties
                      Sheng Meng
                      National University of Singapore

16.15pm – 16.30pm  Break @ Staff Lounge

16.30pm – 17.30pm  Bicanonical maps of ball quotients
                      JongHae Keum
                      Korea Institute for Advanced Study (KIAS)
Abstract

A surface with discrete and non-finitely generated automorphism group

Tien-Cuong Dinh, National University of Singapore

We show that there is a smooth complex projective variety, of any dimension greater than or equal to two, whose automorphism group is discrete and not finitely generated. Moreover, this variety admits infinitely many real forms which are mutually non-isomorphic over R. The talk is based on a joint work with Keiji Oguiso arXiv:1710.07019.

A Lefschetz fibration structure on minimal symplectic fillings of a quotient surface singularity

Jongil Park, Seoul National University

Since it was known that any closed symplectic 4-manifold admits a Lefschetz pencil and that a Lefschetz fibration structure can be obtained from a Lefschetz pencil by blowing-up the base loci, the study of Lefschetz fibrations has become an important research theme for understanding symplectic 4-manifolds topologically. Furthermore, it is also one of active research topics in symplectic 4-manifolds to classify symplectic fillings of certain 3-manifolds equipped with a contact structure.

Among them, people have long studied symplectic fillings of the link of a quotient surface singularity. For example, P. Lisca classified symplectic fillings of cyclic quotient singularities whose corresponding link is lens space, and M. Bhupal and K. Ono classified all possible symplectic fillings of non-cyclic quotient surface singularities. And then, J. Park together with Heesang Park, Dong-soo Shin, and Giancarlo Urzu´a constructed an explicit one-to-one correspondence between the minimal symplectic fillings and the Milnor fibers of non-cyclic quotient surface singularities.

By the way, M. Bhupal and B. Ozbagci found an algorithm to present each minimal symplectic filling of a cyclic quotient sur- face singularity as an explicit positive allowable Lefschetz fibration, called PALF, structure. Furthermore they showed that each PALF structure can be obtained from the minimal resolution by monodromy substitutions which correspond to rational blow-downs topologically. In this talk, I’d like to explain how to construct an explicit PALF structure on any minimal symplectic filling of the link of non-cyclic quotient surface singularities. This is a joint work with Hakho Choi.
Bicanonical maps of ball quotients

**JongHae Keum, Korea Institute for Advanced Study (KIAS)**

Applying I. Reider's theorem, one sees easily that the bicanonical map of a ball quotient is base point free, so defines a morphism. Its very ampleness is a subtle and interesting problem, especially for ball quotients with small Euler characteristic, e.g., fake projective planes and the Cartwright-Steger surface. I will talk on what has been known.

Threefold in \( \mathbb{P}^5 \): multisecant and regularity

**Sijong Kwak, Korea Advanced Institute of Science and Technology (KAIST)**

Eisenbud-Goto regularity conjecture has been considered as true for a long time and tried to prove it. However, J. McCullough and Peeva constructed many counterexamples for singular integral varieties. So, there is a mysterious dichotomy between smooth and singular cases. For positive answers so far, the loci of multisecant lines, e.g. classical trisecant lines for curves, \((\text{dimension}+2)\)-secant lemma due to Ran and more general theorem on multisecants due to Eisenbud-Beheshti have important roles. In the case of threefolds of codimension 2, we explain why the Eisenbud-Goto conjecture is true for smooth case and also introduce counterexamples for singular threefold cases.

On polarized and amplified endomorphisms of normal projective varieties

**Sheng Meng, NUS**

Let \( X \) be a normal projective variety over an algebraically closed field \( k \) of characteristic 0. We consider a non-isomorphic polarized endomorphism \( f \) of \( X \), that is \( f^*L \) is linearly equivalent to \( qL \) for some ample Cartier divisors \( L \) and \( q>1 \).

In this talk, we’ll first give a rough characterization of \( X \) related to its singularities, canonical divisor, MRC fibration and Albanese map, etc. We’ll then show that one can run the minimal model program (MMP) \( f \)-equivariantly, after replacing \( f \) by a positive power, for a mildly singular \( X \). In the end, we’ll generalize part of the above results to the case of positive characteristic and the case of int-amplified \( f \), that is \( f^*L-L = H \) for some ample Cartier divisors \( L \) and \( H \).

Some results are joint works with Paolo Cascini and De-Qi Zhang