

Geometric structures and representation varieties workshop

3-5 May 2017

Department of Mathematics and IMS
National University of Singapore

Program

Wednesday, May 3, 2017

NUS mathematics department, SR1 (seminar room 1)

- 2-3pm: **G rard Besson**
Some open subsets of S^3 (also math dept colloquium)
- 4-5pm: **Graeme Wilkin**
The reverse Yang-Mills-Higgs flow in the neighbourhood of a critical point
- 6.30: Social dinner at the Scholar restaurant, NUSS.

Thursday, May 4, 2017

NUS mathematics department, SR1 (seminar room 1)

- 10-11am: **Ser Peow Tan**
Hyperbolic jigsaws and families of pseudomodular groups
- 11:15-12:15: **Olivier Glorieux**
Hausdorff dimension and critical exponent of Quasi-Fuchsian Anti-de Sitter manifolds
- 2:30-3:30pm: **Andrew Yarmola**
Hyperbolic 3-manifolds with low cusp volume
- 4-5pm **Greg McShane**
Sets of Geodesics and commensurability
- 6.30: Conference dinner (Venue TBA)

Friday, May 5, 2017

IMS auditorium

- 9:30-10:30am: **Sadayoshi Kojima**
On the moduli space of equilateral plane pentagons
- 10:45-11:45 **Inkang Kim**
Gromov Simplicial volume, Barycenter method, and Bounded cohomology
- 12:00-13:30 **Lunch at IMS**
- 2:00-3:00pm: **Cl ment Gu rin**
The Fricke space of the 1-holed Klein bottle
- 3:30-4:30pm: **Hugo Parlier**
Interrogating length spectra and quantifying isospectral finiteness

END OF WORKSHOP

ABSTRACTS

G rard Besson

Some open subsets of S^3 (also math dept colloquium)

We will describe in a very elementary way the geometry and topology of some open subsets of S^3 . They are open 3-manifolds, some with very interesting topology. The geometry is much less understood and we will state some of the known results and address interesting questions in geometry as well as in analysis.

Olivier Glorieux

Hausdorff dimension and critical exponent of Quasi-Fuchsian Anti-de Sitter manifolds

The aim is to explain how classical invariants and theorems in the hyperbolic setting can be extended to the Anti-de Sitter (AdS) setting. We are first going to explain how we can define a good notion of growth for certain groups acting on AdS manifolds. Then how we can extend the notion of Hausdorff dimension of limit set on the boundary of AdS. We will finally explain how to get a rigid bound for these invariants in dimension 3. This is a joint work with D. Monclair.

Cl ment Gu rin

The Fricke space of the 1-holed Klein bottle

The Fricke space of the 1-holed Klein bottle can be identified to a subset of a real form of the character variety of a free group F_2 of rank 2 into $SL(2, \mathbb{C})$. In their article (arXiv : 1509.03790), Goldman, McShane, Stantchev and Tan study the action of automorphisms of F_2 on this real form. They prove that the Fricke space of the 1-holed Klein bottle is dense in this real form on each level set for the commutator trace. I will explain why it should be expected that the complementary is of measure zero.

Inkang Kim

Gromov Simplicial volume, Barycenter method, and Bounded cohomology

We prove that the locally finite simplicial volume of most Q -rank 1 locally symmetric spaces is positive. We combine the method of Gromov's bounded cohomology theory, Besson-Courtois-Gallot and Connell-Farb's barycenter method, and symmetric space geometry.

Sadayoshi Kojima

On the moduli space of equilateral plane pentagons

This is a joint work with Stephan Klaus at MFO. We give a new proof of the known result that the moduli space of equilateral plane pentagons is a closed surface of genus 4. Moreover, we provide a new algebraic description of this space, also in the non-equilateral case, as a real affine algebraic surface defined by a polynomial of degree 12. This allows a visualization using the Surfer software.

Greg McShane

Sets of Geodesics and commensurability

Let Γ be a Fuchsian group. We denote by $\text{ax}(\Gamma)$ the set of axes of hyperbolic elements of Γ . Define Fuchsian groups Γ_1 and Γ_2 to be isoaxial if $\text{ax}(\Gamma_1) = \text{ax}(\Gamma_2)$. Reid and Long proved the following theorem.

Theorem. Let Γ_1 and Γ_2 be isoaxial **arithmetic** Fuchsian groups. Then Γ_1 and Γ_2 are commensurable.

We will discuss the general case in particular showing that generically $\text{ax}(\Gamma)$ determines the commensurability class of Γ . We will also show that the Birman Series set determines a closed surface up to isometry.

Hugo Parlier

Interrogating length spectra and quantifying isospectral finiteness

Associated to a closed hyperbolic surface is its length spectrum, the set of the lengths of all of its closed geodesics. Two surfaces are said to be isospectral if they share the same length spectrum. There are different methods to produce surfaces that are isospectral but not isometric, the most successful one based on a technique introduced by Sunada.

The talk will be about the following questions and how they relate:

How many questions do you need to ask a length spectrum to determine it?

Among all surfaces of given genus, how many can be isospectral but not isometric?

The approach to these questions will include finding adapted coordinate sets for moduli spaces and exploring McShane type identities.

Ser Peow Tan

Hyperbolic jigsaws and families of pseudomodular groups

It is well known that the modular group $\text{PSL}(2, \mathbb{Z})$ has cusp set all of the rationals. A Fuchsian group which has cusp set all of the rationals but which is not commensurable to $\text{PSL}(2, \mathbb{Z})$ is called a pseudomodular group. Long and Reid found some examples of pseudomodular groups (4 commensurability classes, in fact) and asked if there were infinitely many commensurability classes of pseudomodular groups. In this talk we will describe a way to construct Fuchsian groups by a hyperbolic jigsaw construction and answer the question in the affirmative. This is joint work with Beicheng Lou and Anh Duc Vo.

Graeme Wilkin

The reverse Yang-Mills-Higgs flow in the neighbourhood of a critical point

The Yang-Mills-Higgs flow was originally studied by Simpson in the context of constructing Hermitian-Einstein metrics on Higgs bundles. Methods of Donaldson and Simpson show that the Yang-Mills-Higgs flow resembles a nonlinear heat equation on the space of Hermitian metrics on the bundle and therefore the downwards flow is well-behaved with respect to existence and uniqueness, and it also has nice smoothing properties. On the other hand, the reverse flow is ill-posed and therefore even existence of solutions is not guaranteed.

In this talk I will describe a new method to construct long-time solutions to the reverse Yang-Mills-Higgs flow that converge to a given critical point. The methods naturally lead to an algebraic criterion for two critical points to be connected by a flow line and a geometric condition to distinguish between broken and unbroken flow lines in terms of secant varieties of the underlying curve. I will then show how the same ideas can be applied to moment map flows on the space of representations of a quiver with relations, and that this can be used to develop a Morse theory on this space.

Andrew Yarmola

Hyperbolic 3-manifolds with low cusp volume

For a cusped hyperbolic 3-manifold, one can consider the volume of the maximal horoball neighborhood of a cusp. In this talk, we will present preliminary results for classifying the infinite families of hyperbolic 3-manifolds of cusp volume < 2.62 and the implications of this classification. These families are of particular interest as they exhibit the largest number of exceptional Dehn fillings. Our classification also gives a direct path to classify the first 3 smallest volume closed hyperbolic manifolds. As in some other results on hyperbolic 3-manifolds of low volume, our technique utilizes a rigorous computer assisted search. This talk will focus on providing sufficient background to explain our approach and describe our conclusions. This work is joint with David Gabai, Robert Meyerhoff, Nathaniel Thurston, and Robert Haraway.