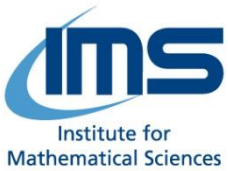
Workshop on

Nonlinear PDEs and Applications

25 September 2013 (Wednesday)
S17 #04-06

Jointly organized by

Department of Mathematics
Faculty of Science



Programme

9.00am – 9.50am	**Quantum Hall Hases and Plasma Analogy in Rotating Bose Gases** *Jakob Yngvason (University of Vienna, Austria)*
9.50am – 10.20pm	Break @ Mathematics Department Lounge
10.20am – 11.10am	**The Strichartz Inequality for Orthonormal Functions** *Robert Seiringer (Institute of Science and Technology, Austria)*
11.10am – 12.00pm	**Modeling Barrier-Crossing Events in Complex Systems** *Weiqing Ren (National University of Singapore, Singapore)*
12.00pm – 2.00pm	Lunch @ Mathematics Department Lounge
2.00pm – 2.50pm	**Lieb-Thirring inequalities for Anyons** *Jan Philip Solovej (University of Copenhagen, Denmark)*
2.50pm – 3.20pm	Break @ Mathematics Department Lounge
3.20pm – 4.10pm	**Analysis and Computation for the Semiclassical Limits of the Nonlinear Schroedinger Equations** *Weizhu Bao (National University of Singapore, Singapore)*
4.10pm – 5.00pm	**Order of Convergence of Splitting Schemes for Both Deterministic and Stochastic Nonlinear Schroedinger Equations** *Jie Liu (National University of Singapore, Singapore)*

For more information, please contact
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Abstracts

Analysis and Computation for the Semiclassical Limits of the Nonlinear Schroedinger Equations

Weizhu Bao, National University of Singapore, Singapore

In this talk, I will review recent results on analysis and efficient computation for the semiclassical limits of linear and nonlinear Schroedinger (NLS) equations. First, I will show our recent asymptotic and numerical results on the semiclassical limits of the ground and excited states of time-independent NLS with a few typical external trapping potentials. Then I will review the formal semiclassical limit of the NLS by using different approaches including WKB method, Winger transform, Grenier's generalized WKB analysis, etc. A time-splitting spectral (TSSP) method was introduced to efficiently compute the dynamics of the NLS in the semiclassical regimes. The numerical method is explicit, unconditionally stable, time reversible and time transverse invariant. Moreover, it conserves the position density in the discretized level and has the best spatial/temporal resolution for the NLS in the semiclassical regimes. Comparison between the solutions of the NLS and its quantum hydrodynamical limit are presented, especially when the quantum hydrodynamical equations have shocks and/or vacuum. Finally, the analysis and computation results are extended for the NLS with an angular momentum rotation term and coupled nonlinear Schroedinger equations.

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Order of Convergence of Splitting Schemes for both Deterministic and Stochastic Nonlinear Schroedinger Equations

Jie Liu, National University of Singapore, Singapore

We first prove the second order convergence of the Strang-type splitting scheme for the nonlinear Schrodinger equation. The proof does not require commutator estimates but crucially relies on an integral representation of the scheme. It reveals the connection between Strang-type splitting and the midpoint rule. We then show that the integral representation idea can also be used to study the stochastic nonlinear Schrodinger equation with multiplicative noise of Stratonovich type. Even though the nonlinear term there is not globally Lipschitz, we prove the first order convergence of a splitting scheme of it. Both schemes preserve the mass. They are very efficient because they use explicit formulas to solve the subproblems containing the nonlinear or the nonlinear plus stochastic terms.

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Modeling Barrier-Crossing Events in Complex Systems
Weiqing Ren, National University of Singapore, Singapore

The dynamics of complex systems is often driven by rare but important events. Well known examples include nucleation events during phase transitions, conformational changes of macro-molecules, chemical reactions, etc. The long time scale associated with these rare events is a consequence of the disparity between the effective thermal energy and typical energy barrier of the systems. The dynamics proceeds by long waiting periods around metastable states followed by sudden jumps from one state to another. The disparity of time scales makes the study of such rare event a very challenging task. In this talk, we discuss the string method for the study of rare events and its extension for saddle point search.

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The Strichartz Inequality for Orthonormal Functions
Robert Seiringer, Institute of Science and Technology, Austria

We prove a Strichartz inequality for a system of orthonormal functions, with an optimal behavior of the constant in the limit of a large number of functions. The estimate generalizes the usual Strichartz inequality, in the same fashion as the Lieb-Thirring inequality generalizes the Sobolev inequality. As an application, we consider the Schroedinger equation in a time dependent potential and we show the existence of the wave operator in Schatten spaces. (This is joint work with Rupert Frank, Mathieu Lewin, and Elliott Lieb.)

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Lieb-Thirring inequalities for Anyons
Jan Philip Solovej, University of Copenhagen, Denmark

I will discuss identical particles in 2-dimensions. In 3 and higher dimensions there are two types of identical particle fermions and bosons.

As is well known they have very different thermodynamic properties. Free fermions have a degeneracy pressure due to the Pauli exclusion principle.

Bosons do not have a degeneracy pressure. The Lieb-Thirring inequality is an elegant expression of the degeneracy pressure. In 2 dimensions there are particles of intermediate statistics described by a statistics parameter running from 0 to 1 (0 being bosons and 1 being fermions). I will show that free anyons satisfy a Lieb-Thirring inequality if the statistics parameter is rational with odd denominator. This is joint work with Douglas Lundholm.

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Quantum Hall Phases and Plasma Analogy in Rotating Bose Gases

Jakob Yngvason, University of Vienna, Austria

A bosonic analogue of the fractional quantum Hall effect occurs in rapidly rotating trapped Bose gases: As the parameters of the Hamiltonian are varied there is a transition from uncorrelated Hartree states to strongly correlated states such as the Laughlin wave function. This physics may be described by effective Hamiltonians with delta interactions acting on a bosonic N-body Bargmann space of analytic functions. I shall report on some joint work with Nicolas Rougerie and Sylvia Serfaty on the transition to strongly correlated states in a trapping potential that is a sum of a quadratic and a quadratic term. Our analysis is based on the interpretation of the densities of quantum Hall trial states as Gibbs measures of classical 2D Coulomb gases (plasma analogy).

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