WORKSHOP ON

Partial Differential Equations and Scientific Computing

15 December 2006

S14 #03-10 (CRA)

Department of Mathematics

National University of Singapore

NUS

National University of Singapore

PROGRAMME

Chair: Weizhu Bao (NUS)

8.55am	-	9.00am	Opening address *Chi Tat Chong* (Head, Department of Mathematics, NUS)
9.00am	-	9.40am	Shock Wave Theory *Taiping Liu* (Academia Sinica, Taiwan and Stanford University, USA)
9.40am	-	10.20am	Fourth Order Non-Linear Equation On 4-Torus *Xingwang Xu (NUS)*
10.20am	-	10.50am	Break

Chair: Tiegang Liu (IHPC)

10.50am	-	11.30am	A New Spectral-Galerkin Method for High-Dimensional PDEs: algorithms, Analysis and Applications *Jie Shen* (Purdue University, USA)
11.30am	-	12.10pm	Efficient and Stable Numerical Methods for Zakharov System *Weizhu Bao* (NUS)
12.10pm	-	2.00pm	Lunch

Chair: Xingwang Xu (NUS)

2.00pm	-	2.40pm	Highly Oscillatory Quadrature and Initial-Value Integrators *Arieh Iserles* (Cambridge University, UK)
2.40pm	-	3.20pm	Coarse-Graining, Approximation and Simulations of Stochastic Systems *Peter Plechac* (University of Warwick, UK)
3.20pm	-	3.50pm	Break

Chair: Ping Lin (NUS)

3.50pm	-	4.30pm	Glass Flow and Applications *Huaxiong Huang* (University of York, Canada)
4.30pm	-	5.10pm	The Modified Ghost Fluid Method for Treating Moving Interfaces and its Applications *Tiegang Liu* (Institute of High Performance Computing, Singapore)
5.10pm	-	6.00pm	Tour of Novel Prize Exhibition at University Hall

For more information, please contact:
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ABSTRACTS

Efficient and Stable Numerical Methods for the Zakharov System
Weizhu Bao, National University of Singapore, Singapore

In this talk, we present efficient and stable numerical methods for the generalized Zakharov system (GZS) describing the propagation of Langmuir wavesin plasma. The key point in designing the methods is based on a time-splitting discretization of a Schroedinger-type equation in GZS, and to discretize a nonlinear wave-type equation by pseudospectral method for spatial derivatives, and then solving the ordinary differential equations in phase space analytically under appropriate chosen transmission conditions between different time intervals or applying Crank-Nicolson/leap-frog for linear/nonlinear terms for time derivatives. The methods are explicit, unconditionally stable, of spectral-order accuracy in space and second-order accuracy in time. Moreover, they are time reversible and time transverse invariant if GZS is, conserve the wave energy as that in GZS, give exact results for the plane-wave solution and possesses `optimal' meshing strategy in `subsonic limit' regime. Extensive numerical tests are presented for plane waves, solitary-wave collisions in 1D of GZS and 3D dynamics of GZS to demonstrate efficiency and high resolution of the numerical methods.

Finally the methods are extended to vector Zakharov system for multi-component plasma and Maxwell-Dirac system (MD) for time-evolution of fast (relativistic) electrons and positrons within self-consistent generated electromagnetic fields.

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Glass Flow and Applications
Huaxiong Huan, University of York, Canada

Appearing as a solid, glass behaves more like a liquid, especially when it is heated. This is largely due to the way glass molecules are connected to each other (random instead of ordered). This fluid-like behavior has been exploited by engineering in manufacturing glass products/devices (anyone who has been to a glassware workshop knows when he/she watches beautiful glass vases being made). In this talk we focus on two specific applications: drawing optical fibers and making glass microelectrodes. We will present simplified mathematical models based on long wave approximation and show that asymptotic solutions can be used for investigating various interesting phenomena as well as addressing relevant practical issues such as how to control the electrode shape or the refraction index in optical fibers.

This is joint work with J. Wylie (City U of HK), R. Miura (NJIT) and P. Howell (Oxford).

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ABSTRACTS

Highly Oscillatory Quadrature and Initial-Value Integrators
Arieh Iserles, Cambridge University, UK

In this lecture we review recent developments in the general area of highly oscillatory quadrature: asymptotic expansions, Filon-type and Levin-type methods and the method of numerical stationary phase. The exposition will cover both univariate and multivariate integrals and consider the important (and difficult) special case of critical points. This theory will be applied to the numerical solution of initial-value ordinary differential equations that exhibit high oscillation, using Magnus and Neumann expansions and exponential integrators.

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Shock Wave Theory
Tai-Ping Liu, Academia Sinica, Taipei and Stanford University, USA

The shock wave theory for one space dimension has seen tremendous progresses for the second half of 20th century. There are now well-posedness as well as regularity and time-asymptotic properties for general solutions. Starting around the 21st century, serious efforts start to yield results on multi-dimensional gas flows with shocks. Classical problems posed during the first half of 20th century are studied and some of them have even been solved. We will describe these and raise possible future research directions.

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The Modified Ghost Fluid Method for Treating Moving Interfaces and Its Applications
Tiegang Liu, Institute of High Performance Computing, Singapore

In this talk, I introduce the Ghost Fluid Method (GFM)—a latest technique to treat moving flow or material interface. It has been found by us that the original Ghost Fluid Method (GFM) may provide incorrect results or even fails to work in some situations. A modified Ghost Fluid Method (MGFM) has been developed by us to overcome the difficulties encountered by the original GFM. I have theoretically shown that the original GFM has no order accuracy and the MGFM has second order accuracy. In this talk, I shall analyze the conservative errors on both the original GFM and the MGFM. Finally, I will give various applications of the MGFM.

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Coarse-Graining, Approximation and Simulations of Stochastic Systems
Peter Plechac, University of Warwick, UK

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ABSTRACTS

A New Spectral-Galerkin Method for High-Dimensional PDEs: Algorithms, Analysis and Applications

Jie Shen, Purdue University, USA

Many scientific, engineering and financial applications require solving high-dimensional PDEs. However, traditional tensor product based algorithms suffer from the so called "curse of dimensionality". We shall present a new Chebyshev-Galerkin method which is based on two basic ingredients: (i) Choosing the frequencies of the trial functions from the "hyperbolic cross"; (ii) Using a lattice rule to perform the integration. It is shown that with this combination, the "curse of dimensionality" can be broken to some extent. We shall present rigorous error estimates and numerical results supporting this statement.

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Fourth Order Non-Linear Equation on 4-Torus

Xingwang Xu, National University of Singapore, Singapore

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