INSTRUCTIONS TO CANDIDATES

1. This paper contains a total of EIGHT (8) questions and comprises FOUR (4) printed pages.

2. Attempt all questions.

3. Non-programmable scientific calculators may be used. However, candidates should lay out systematically the various steps in the calculations.
Question 1 [10 marks]
Express in partial fractions
\[ \frac{x + 5}{(x^2 + 1)(3 - 2x)}. \]
Hence, find the exact value of
\[ \int_0^1 \frac{x + 5}{(x^2 + 1)(3 - 2x)} \, dx. \]

Question 2 [10 marks]
Prove by induction that
\[ \sum_{r=0}^{n-1} \cos(a + 2r\beta) = \frac{\cos(a + (n-1)\beta)\sin n\beta}{\sin \beta} \]
where \(\sin \beta \neq 0\)
Deduce that
\[ \cos \theta + \cos 5\theta + \cos 9\theta + \cos 13\theta = \frac{\cos 7\theta \sin 8\theta}{\sin 2\theta}, \]
where \(\sin 2\theta \neq 0\).

Question 3 [10 marks]
The parametric equations of a curve are \(x = a(t + t^2)\) and \(y = a(1 + t\ln t)\), where \(a\) is a constant and \(t > 0\).
(i) Find the equation of the normal to the curve at the point where \(t = 1\).
(ii) Obtain and simplify an expression for \(\frac{d^2 y}{dx^2}\).

Question 4 [10 marks]
The function \(f\) is defined by
\[ f(x) = \frac{1}{6}x^3 - \frac{36}{x}, \quad x > 0. \]
(i) Show that the function \(f\) has an inverse.
(ii) The graphs of \(f\) and its inverse meet at the point A. Find the coordinates of A.
**Question 5 [10 marks]**

Express the complex number \( z = \frac{(1 - \sqrt{3}i)^4}{(1 + i)^2} \) in the form \( r(\cos \theta + i \sin \theta) \), where \( r > 0 \) and \( -\pi < \theta \leq \pi \).

On an Argand diagram, the point P represents the complex number \( z \) and the point Q represents the complex number \( w \). Given that triangle OPQ oriented in the clockwise sense is equilateral, find the modulus and argument of \( w \).

**Question 6 [15 marks]**

(a) Show that \( \frac{d}{dx} \left\{ \ln\left(x + \sqrt{x^2 + 1}\right) + x\sqrt{x^2 + 1} \right\} = 2\sqrt{x^2 + 1} \).

Hence, find \( \int \frac{(x+1)^2}{\sqrt{x^2 + 1}} \, dx \).

(b) Evaluate \( \int \frac{(x+1)}{e^{x^2}} \, dx \).

**Question 7 [15 marks]**

*For this question, give your answer to three significant figures where appropriate.*

(a) If \( \cos 2x, \sin 4x, \cos 6x \) are successive terms of an arithmetic progression and \( -\frac{\pi}{2} < x < \frac{\pi}{2} \), find all possible values of \( x \).

(b) If \( \sqrt{6 - 2^y}, 4^y, \sqrt{6 + 2^y} \) are successive terms of a geometric progression, find all possible values of \( y \).
Question 8 [20 marks]

The curve C has equation

\[ y = \left| \frac{x - 9}{x} \right|, \quad x \neq 0. \]

(i) Find the coordinates of the points where the curve meets the \( x \)-axis.

(ii) Show that the curve has no stationary points.

(iii) Sketch the curve C.

(iv) Find the range of values of \( x \) for which the curve lies above the line \( y = 8 \).

(v) Let \( R \) be the region in the first quadrant bounded by the curve C and the line \( y = 8 \). Calculate the exact volume of the solid formed when the region \( R \) is rotated completely about the \( x \)-axis.