INSTRUCTIONS TO CANDIDATES

1. This paper contains a total of **EIGHT (8)** questions and comprises **FIVE (5)** printed pages.

2. Attempt all questions.

3. Non-programmable scientific calculators may be used. However, candidates should lay out systematically the various steps in the calculations.
Question 1  [14 marks]

(a) Sketch, on the same diagram, the graphs of \( y = 3x^2 + 1 \) and \( y = |x - 3| \).

Hence, solve the inequality

\[ 3x^2 + 1 > |x - 3|. \]

(b) Use the Factor Theorem to find the linear factors of \( 2x^3 - 9x^2 - 2x + 24 \).

Hence, solve the inequality

\[ \frac{1}{2x^3 - 9x^2 - 2x + 24} \leq 0. \]

Question 2  [8 marks]

The functions \( f \) and \( g \) are given by

\[ f : x \rightarrow 3 - 4x - x^2, \quad x < c. \]
\[ g : x \rightarrow \ln(3 - x), \quad x < 3. \]

(i) Find an expression for \( g^{-1}(x) \).

(ii) Find the range of the function \( f \) when \( c = 0 \).

(iii) Find the largest value of \( c \) for which the composite function \( gf \) is defined.

Question 3  [12 marks]

(a) It is given that \( 3^x, 2^{x+1}, 4^{x-1} \) are the first three terms of a geometric progression.

(i) Find, to three significant figures, the value of \( x \).

(ii) Find, to the nearest integer, the sum to infinity of the progression.

(b) Given that \( \sum_{r=1}^{n} r = \frac{1}{2} n(n + 1) \) and \( \sum_{r=1}^{n} r^2 = \frac{1}{6} n(n + 1)(2n + 1) \), find in terms of \( n \)

\[ \sum_{r=0}^{3n} r(3n - r). \]
**Question 4  [12 marks]**

Let \( z = \sqrt{3} e^{\frac{i\pi}{6}} \) and \( w = -2 - 2i \).

(i) Express \( w \) in the form \( r(\cos \theta + i \sin \theta) \), where \( r > 0 \) and \( -\pi < \theta \leq \pi \).

(ii) Find the modulus of \( \frac{z^6}{w^4} \).

(iii) Find the argument of \( z^* w^2 \), where \( z^* \) denotes the conjugate of \( z \).

(iv) In an Argand diagram, the points \( A, B \) and \( C \) represent the complex numbers \( z, w \) and \( v \) respectively, and \( O \) is the origin. OACB is a parallelogram described in the clockwise sense. Find the real part of \( v \).

**Question 5  [12 marks]**

(a) Express \( \frac{x^2 - 4x - 27}{(x - 1)(x^2 + 9)} \) in partial fractions.

Hence, find

\[
\int \frac{x^2 - 4x - 27}{(x - 1)(x^2 + 9)} \, dx.
\]

(b) Find the exact value of

\[
\int_0^2 (2x + 1)e^{2x} \, dx.
\]
**Question 6**  [12 marks]

(a) A curve is defined by the parametric equations

\[
x = t + \frac{1}{2} \sin 2t, \quad y = 2 \sin t + \frac{1}{2} \sin 2t
\]

for \(-\frac{\pi}{2} < t < \frac{\pi}{2}\).

(i) Find \(\frac{dx}{dt}\) and \(\frac{dy}{dt}\).

(ii) Show that \(\frac{dy}{dx} = 1 + \sec t\).

(b) Given that \(2x^3 + 3e^x + 4xy = 5\), find \(\frac{dy}{dx}\) in terms of \(x\) and \(y\).

**Question 7**  [15 marks]

In the diagram, the curve \(y = 16 - x^4\) meets the \(x\)-axis and \(y\)-axis at the points A and B respectively. P is a variable point on the arc AB and Q is the foot of perpendicular from P to the \(x\)-axis. O is the origin.

(i) Find the coordinates of A and B.

(ii) Find the equation of the tangent to the curve at the point A.

(iii) Show that the area of trapezium \(OBPQ\) is \(\frac{1}{2}(32p - p^5)\).

Hence, determine the maximum area of trapezium \(OBPQ\) as \(p\) varies. Give your answer to three significant figures.
In the diagram, the shaded region labelled R is bounded between the curve \( y = \frac{15}{\sqrt{25-x^2}} \) and the chord joining the points A\((0, 3)\) and B\((4, 5)\).

(i) Find the equation of the straight line passing through the points A and B.

(ii) Calculate the exact area of the region R.

(iii) The region R is rotated completely about the y-axis. Find the exact volume of the solid formed.