INSTRUCTIONS TO CANDIDATES

1. This paper contains a total of **EIGHT (8)** questions and comprises **FOUR (4)** printed pages.

2. Attempt all questions.

3. Non-programmable scientific calculators may be used. However, candidates should lay out systematically the various steps in the calculations.
Question 1 [12 marks]

(a) Find the coefficient of $x^{10}$ in the expansion, in ascending powers of $x$, of $\frac{4}{2-3x^2}$.

(b) Find the coefficient of $x^{-10}$ in the expansion of $\left(x-\frac{3}{2x^2}\right)^8$.

Question 2 [12 marks]

(i) Factorise $2x^3 + 5x^2 + 4x + 1$ completely.

(ii) Prove by induction that

\[
\sum_{r=1}^{n} \frac{2r^2 - 1}{(2r+1)(2r-1)} = \frac{n^2}{2n+1}.
\]

Question 3 [12 marks]

The functions $f$ and $g$ are defined by

\[ f : x \rightarrow 10 - x^3, \quad 0 \leq x < a, \]

\[ g : x \rightarrow e^{\sqrt{x-2}}, \quad x > 2. \]

where $a$ is a positive constant.

(i) Find the range of values of $a$ for which the composite function $gf$ is defined.

(ii) Find an expression for $g^{-1}(x)$, where $g^{-1}$ denotes the inverse of $g$.

(iii) Find the value of $x$ for which $f^{-1}(x) = f(x)$, where $f^{-1}$ denotes the inverse of $f$.

Question 4 [12 marks]

(a) Find the gradient of the curve $y^2 - xy + 2x^3 = 4$ at the point $(1, 2)$.

(b) Find the equation of the tangent to the curve $y = \frac{2\cos 2x}{2 + \sin 2x}$ at the point $(0, 1)$. 
Question 5  [12 marks]
An arithmetic progression has first term  \( a \) and common difference \( d \). The sum of the first 100 terms is \( S \) and the sum of the first 50 even-numbered terms, i.e. the second, forth, sixth, …, hundredth, is \( T \).

(i) Express \( S \) and \( T \) in terms of \( a \) and \( d \).

It is given that \( \frac{S}{T} = \frac{35}{18} \).

(ii) Express \( a \) in terms of \( d \).

(iii) Show that the first, seventeenth and twenty-fifth terms are consecutive terms of a geometric progression.

Question 6  [15 marks]

(a) Express \( \frac{5x^2 - 4x + 2}{x(x^2 + 1)} \) in the form

\[
\frac{A}{x} + \frac{B}{x^2 + 1} + \frac{Cx}{x^2 + 1},
\]

where the numerical values of \( A, B \) and \( C \) are to be found.

Hence, find

\[
\int \frac{5x^2 - 4x + 2}{x(x^2 + 1)} \, dx.
\]

(b) Using the identity

\[
\left(\sin^2 x + \cos^2 x\right)^2 = 1,
\]

or otherwise, express \( \sin^4 x + \cos^4 x \) in the form \( P \cos 4x + Q \), where the numerical values of \( P \) and \( Q \) are to be found.

Hence, find the exact value of

\[
\int_0^{\pi/4} \sin^4 x + \cos^4 x \, dx.
\]
Question 7  [10 marks]
(a) Express \((1 - i)^4(1 + i)^5\) in the form \(r(\cos \theta + i \sin \theta)\), where \(r > 0\) and \(-\pi < \theta \leq \pi\).

(b) Find the complex number \(z\) for which
\[
(3 + 2i)z = 2z^* + 9,
\]
where denotes the conjugate of \(z\).

Question 8  [15 marks]

As shown in the above diagram, the region \(R\) is enclosed by the curve \(y = e^{2x} - 3x\), \(0 \leq x \leq 1\), and the line segment joining the points on the curve whose \(x\)-coordinates are 0 and 1. The region \(S\) is bounded by the curve, the axes and the line \(x = 1\).

Find

(i) the \(x\) – coordinates of the stationary point on the curve.
(ii) the exact area of the region \(R\).
(iii) the exact volume of revolution formed when the region \(S\) is rotated completely about the \(x\)-axis.

END OF PAPER