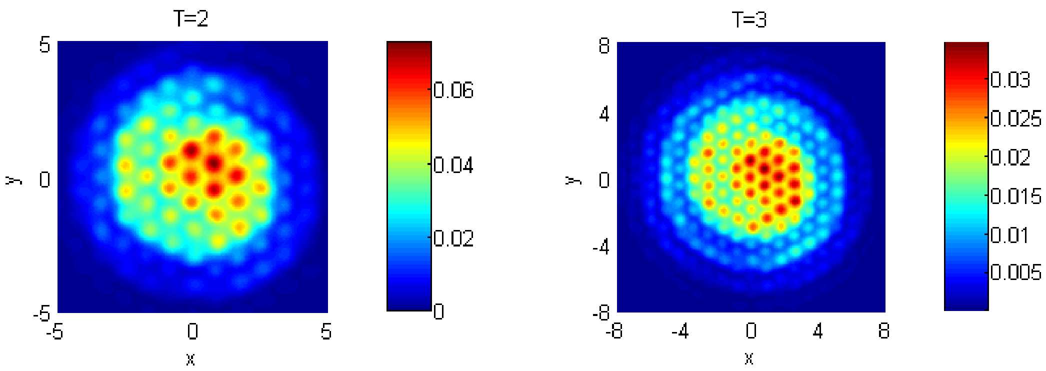
Research Highlight: Multiscale Methods and Analysis for the Dirac Equation

Work of Professor BAO Weizhu

Prof BAO Weizhu and co-authors proposed and analysed multiscale methods for the Dirac equation in the nonrelativistic limit regime. The Dirac equation is a relativistic wave equation derived by the British physicist Paul Dirac in 1928 for the description of elementary spin-1/2 massive particles, such as electrons and positrons. It is consistent with both the principle of quantum mechanics and the theory of special relativity.

In the nonrelativistic limit regime, the solution of the Dirac equation exhibits highly oscillatory propagation waves in time. By adapting the energy method, error bounds were established for the finite difference time domain methods and exponential wave integrator Fourier pseudospectral method. By adopting a multiscale decomposition of the solution of the Dirac equation and applying the exponential wave integrator with appropriate numerical quadrature, a uniformly accurate multiscale time integrator Fourier pseudospectral (MTI-FP) method was proposed and analysed for the Dirac equation [1]. Rigorous error bounds were established, which show that the MTI-FP method converges uniformly and optimally in space with exponential convergence rate if the solution is smooth, and uniformly in time with linear convergence for all wave velocity regime with respect to the speed of light. The MTI-FP method can be applied to simulate the dynamics of the Dirac equation with applications in relativistic quantum mechanics, graphene and graphite as well as two dimensional materials, chiral confinement of Bose-Einstein condensation, neutron interaction in nuclear physics and attosecond laser on molecule.



Density of the Dirac equation in two dimensions with a honeycomb potential

Reference:

[1] W. Bao, Y. Cai, X. Jia and Q. Tang, “A uniformly accurate (UA) multiscale time integrator pseudospectral method for the Dirac equation in the nonrelativistic limit regime”. SIAM Journal on Numerical Analysis, 54 (2016): 1785-1812.

[2] W. Bao, Y. Cai, X. Jia and J. Yin, “Error estimates of numerical methods for the nonlinear Dirac equation in the nonrelativistic limit regime”. Science China Mathematics, 59 (2016): 1461-1494.

[3] W. Bao, Y. Cai, X. Jia and Q. Tang, “Numerical methods and comparison for the Dirac equation in the nonrelativistic limit regime”. Journal of Scientific Computing, to appear (arXiv: 1504.02881).