Research Highlight: Stealthy hyperuniform processes: maximal rigidity and the bounded holes conjecture

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We investigate translation invariant stochastic processes on a Euclidean space or on a Euclidean lattice whosediffraction spectrum or structure function S(k), i.e. the Fourier transform of the truncated total pair correlation function, vanishes on an open set U in the wave space. A key family of such processes are "stealthy" hyperuniform point processes, for which the origin k = 0 is in U; these are of much current physical interest. We show that all such processes exhibit the following remarkable maximal rigidity : namely, the configuration outside a bounded region determines, with probability 1, the exact value (or the exact locations of the points) of the process inside the region. In particular, such processes are completely determined by their tail. In the 1D discrete setting (i.e. Z-valued processes on Z), this can also be seen as a consequence of a recent theorem of Borichev, Sodin and Weiss; in higher dimensions or in the continuum, such a phenomenon seems novel. For stealthy hyperuniform point processes, we prove the Zhang-Stillinger-Torquato conjecture that such processes have bounded holes (empty regions), with a universal bound that depends inversely on the size of U.

Reference:

[1] Ghosh, Subhroshekhar, and Joel L. Lebowitz. "Generalized stealthy hyperuniform processes: maximal rigidity and the bounded holes conjecture." *Communications in Mathematical Physics* 363.1 (2018): 97-110.