Research Highlight : q-Decomposition numbers

Work of Associate Professor TAN Kai Meng

The q-decomposition numbers arising from the canonical basis of the Fock space representation of the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_e)$ of the Lie algebra $\mathfrak{sl}_e$ have been shown to be graded decomposition numbers of the Hecke algebras of symmetric groups, and more generally of v-Schur algebras, at complex e-th root of unity ([BK]). While the canonical basis vectors, and hence the q-decomposition numbers, may be computed in theory, all the algorithms hitherto known are recursive in nature, and may only compute these vectors in 'small cases'.

In [CMT], A/P Tan and his co-authors Joseph Chuang and Hyohe Miyachi use an entirely new and innovative approach to describe closed formulas for a large subset of the canonical basis vectors in terms of tilings of high-dimensional parallelograms, or so-called paralletopes, which assemble to form polytopal complexes. For large e, a randomly chosen canonical basis vector will 'almost surely' lie in this large subset. Furthermore, they show that the 1-skeletons of these polytopal complexes is exactly the subgraph of Ext^1-quiver of the v-Schur algebras with vertices corresponding to the canonical basis vectors lying in this large subset.

References

[BK] J. Brundan, A. Kleshchev, 'Graded decomposition numbers for cyclotomic Hecke algebras', *Advances in Mathematics* **222** (1883–1942), 2009.

[CMT] J. Chuang, H. Miyachi, K. M. Tan, 'Parallelotope tilings and q-decomposition numbers', *Advances in Mathematics* **321** (80–159), 2017.