Boltzmann Equation, Industry Technology Advancement, and Mathematical Analysis

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The milestones in the history of kinetic equations

C. J. Maxwell introduced a velocity distribution function of molecules to compute the bulk viscosity and heat conductivity for gas consistent of hard sphere molecules in 1860, and he showed that the bulk viscosity and heat conductivity are independent of gas density but they depend on the square root of the thermal temperature. In 1872 L. Boltzmann introduced the Boltzmann equation, whose dependent variable is a velocity distribution function of gas molecules, to show the H-theorem which asserts the third law of thermal dynamics, (entropy of a closed system is a monotone increasing function with respect to time), directly from matters and motions.

An experiment by W. Crookes

Radiometer

The industry technology in the era of Queen Victoria was advanced enough to create a highly rarefied environment. In 1873 W. Crookes invented a device which was called the radiometer to verify the Maxwell equations on electromagnetism. At a first glance of the outcome, the device seemed to behave in a way as the Maxwell equations predicted. However, after a closer look at the experiment outcome, the device rotated in an opposite direction, which contradicts to the Maxwell equations prediction. Maxwell made many attempts to explain this anomaly by the conventional fluid mechanics such as the compressible Navier-Stokes equation or compressible Euler Equation, but none of the theories could explain it perfectly. Not until Maxwell received the hint from the Reynolds' attempt to explain the anomaly based on a kinetic equation together with a porous media assumption on the vane of the radiometer in 1879 he did successfully explain the anomaly, which is called the temperature gradient flow, is generic for highly rarefied gas modeled by the Boltzmann equation with a non-small mean free path.

Interests on the rarefied gas in modern times

The development of the space program and star war program in the cold war period raised the topics of rarefied gas as one of the major interests in developing spaceship technology to re-enter the upper layer of the atmosphere and to launch an inter-continental ballistic missile. Those programs encountered a regime where all the traditional theories of the fluid mechanics are not valid. As the industry technology advanced, the needs for the rarefied gas theory increased. For example, the electron flows in a micro device can be viewed as rarefied gas flows since the mean free paths are not small. Another example is that, in the production

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Research Interests

- Boltzmann Equations
- Shock Wave Theories
- Hyperbolic Conservation Laws
- Numerical Computations

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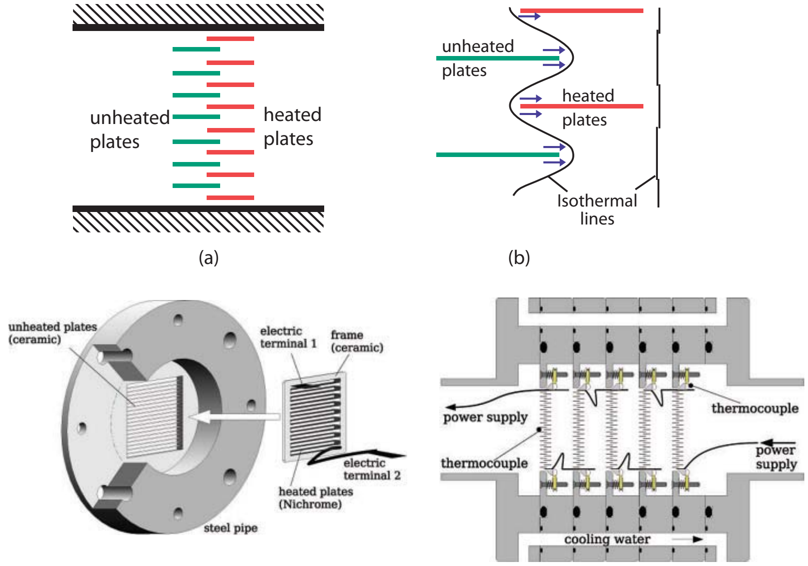
process of semiconductor devices one needs to require a near vacuum environment for the high level of industrial precision. The Boltzmann equation is one of the most used model for the industrial simulation, since the Boltzmann equation is the classical mathematical model for the rarefied gas. The industry technology advancement motivates the research on the Boltzmann equation as well as its mathematical analysis in the past few decades.

Mathematical development

In the sixth of the Hilbert's 21 problems addressed to the International Congress of Mathematicians 1900, Hilbert explicitly mentioned the validity of the Boltzmann equation and its connections to fluid dynamics. Later in 1906, Poincare challenged mathematicians to make sense of kinetic theory, and in 1912 Hilbert used integral operator theory to derive a formal expansion theory to obtain the compressible Euler equation. In 1917 the Chapman-Enskog expansion was derived to obtain the compressible Navier-Stokes equation from the Boltzmann equation, and in 1962 H. Grad developed a general asymptotic theory for the Boltzmann equation. He proposed to include three singular slips, "initial slips, boundary slips, shock slips", in the asymptotic theory. Along with the Grad's proposal, Sone developed the generalized asymptotic theory for boundary value problems, condensation-vaporization problems, etc. Sone, Aoki, and their coworkers in the Engineering School of the Kyoto University obtained a series of high quality asymptotic theories on the temperature gradient flows, condensation-vaporization problem, the ghost effect, etc. Their theories provide good physical understandings of the rarefied gas. With the sharp knowledge, see [3,4], Sone and Sugimoto invented an ingenious device awarded by the Japanese Vacuum Society 2007: a vacuum pump without any moving parts:



Quantitative-Qualitative analysis on the Boltzmann equation

The high quality asymptotic analysis developed by the Sone's group motivated mathematicians to pursue the Boltzmann equation in a classical fashion, i.e. to give a constructive, global, pointwise detailed description of the solution and the coupling of the nonlinear waves. This leads to the consideration of the Green's function for a linearized Boltzmann equation around a thermal-equilibrium state, [1] in which a mathematical mixing phenomenon was discovered: The microscopic regularity can be converted into the macroscopic regularity by mixing information from different locations and different velocities. With this device one can effectively decompose solutions into two dual components: particle-like component and fluid-like components, and this gives two-time scale structures in the solutions, and provides a precise structure of the Green's function. With the sharper estimates on the Green's function one can obtain the detailed global solution of the full Boltzmann equation for a class of small perturbations around a thermal-equilibrium state. With this Green's function as a basic instrument, the Green's function for an initial-boundary value problem was constructed to study the stability of a thermal-equilibrium state for a half-space problem in [2]. More recently with the Green's function as a basic instrument, the nonlinear invariant manifolds for stationary Boltzmann flows were established to yield a series of new theorems on the Boltzmann equation.

Conclusion

The classical mathematical analysis for the Boltzmann equation provides not only the essence of the rarefied gas physics but also a powerful analytical tool. It shall receive further studies and shall devise an effective numerical method to enhance the industry technology advancement, and eventually help mathematics to develop further deep theories

Literature

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