:: Department of **Mathematics**

SUMMARY

The role played by Mathematics in modern society has become even more significant in recent years. Indeed, applications of Mathematics are found not just in the physical and computing sciences and engineering, but also in biological, medical and social sciences, as well as in finance.With the combined capabilities and expertise of 60 faculty members and researchers as well as more than 50 visitors throughout the year, the Department of Mathematics is at the forefront of both fundamental research and the applications of mathematics, covering the most important disciplines in the field of mathematical sciences.

The Department has consistently been ranked among the top 50 mathematics departments in the world, and one of the top in Asia, in studies conducted by various agencies. Having garnered both national and university level awards for its research achievements in both pure and applied research in the last decade, the Department continues to maintain high visibility in numerous research areas, through its publication output in premium journals, participation in the editorial work of major mathematical journals, invitation to give high profile lectures at prestigious conferences, and active collaborations with universities from Asia, Europe and USA.

Amongst the significant research achievements, we highlight two most recent groundbreaking pieces of work by Professor Zhu Chengbo and Professor Yu Shih-Hsien respectively. In collaboration with Sun Binyong of the Chinese Academy of Sciences, Prof Zhu settled the long standing "multiplicity-one conjectures" in representation theory. It is expected that the work will have deep impact on the study of L-functions, a fundamental object of investigation in contemporary mathematics. Prof Yu's work completes the studies on shock profile stability problem which has remained unsolved since 1985.

Research Breakthrough

MATHEMATICS

Multiplicity One in Symmetry Breaking

Professor Zhu Chengbo

$$dim\ \mathrm{Hom}_{G'}(V,V') \leq 1$$

Representation theory is a field in mathematics that involves the study of symmetry patterns. Branching, also known as symmetry breaking, is a process of comparison for two symmetry patterns, one larger than the other. In analyzing any symmetry pattern, it is important to understand how it branches. The situation is particularly favorable if one can show a certain uniqueness property called multiplicity-one, where the larger symmetry pattern breaks into sub-symmetry patterns, each of which appears at most once.

Starting from the 1920's at the time of Cartan and Weyl, there have been huge advances in the understanding of continuous symmetry patterns. A very important development in the area was the discovery by Casselman and Wallach of the canonical nature of the smooth models of such symmetry patterns. The reason for the particular interest in the smooth model derives from the fact that many key objects in number theory can be interpreted as living on smooth models.

Motivated by the study of automorphic L-functions (objects which are ubiquitous in contemporary mathematics and which are supposed to have deep links with all sorts of interesting arithmetic information), Bernstein and Rallis, conjectured in the 1980's that certain classical pairs always have multiplicity-one branching in their smooth models. This is known as multiplicity-one conjectures.

In 2008, jointly with Binyong Sun of the Chinese Academy of Sciences, Professor Zhu Chengbo established the validity of this long-standing conjecture in full. Together with its p-adic analog which was demonstrated by four other mathematicians in 2007, this is expected to have important applications to the study of L-functions, especially their central values as predicted by the well-known conjecture of Gross and Prasad.

INTERVIEW with Professor Zhu Chengbo

http://ww1.math.nus.edu.sg/onepageCV.aspx?id=matzhucb

'Every difficult moment in research is an opportunity to get a better feel of it.'

Professor Zhu Chengbo graduated from Zhejiang University, China in 1984 and received his Ph.D. from Yale University in 1990. He has been with the Department of Mathematics at NUS since 1991. His main research interest is in representation theory (the study of symmetries), and its applications in analysis and number theory. In this interview he shares some fundamentals in his work 'Multiplicity One in Symmetry Breaking", a breakthrough in the mathematical study of symmetry patterns.

What is the motivation behind your research work?

Broadly speaking, in Mathematics there are two types of symmetries: real and p-adic. There is one guiding philosophy in our field, called Harish-Chandra's Lefschetz Principle, which says that whatever is true for real groups should also be true for p-adic groups. In fact for our research project, the multiplicity-one conjecture was made for both real and p-adic groups. There was a breakthrough on this for p-adic groups in 2007 by four mathematicians. We solved the conjecture for real groups in 2008.

What are the practical applications of your research work in the real world?

We use math constantly in our daily lives, mostly through numbers. An obvious area is in business. Another is in the vast amount of data being generated which clearly matter to governments, business, and individuals for decision making. In a less apparent way, we use math to understand the forces of planetary motion, to build theory for DNA sequencing, to transfer information across the globe, and so forth.

Representation theory, the area I work in, actually had one of its origins in the study of heat transfer (theory as developed by Fourier); it got invigorated with the rise of quantum mechanics in the first part of 20th century. Mathematics has a fantastic record in producing highly applicable knowledge and there is every indication that it will remain so.

What are your future plans for research?

My previous research works were basically on "local" studies of symmetries. The "global" studies of symmetries are intimately connected with number theory with which I am less familiar. I intend to expand my current line of research by learning the "global" aspects, which may take several years.

To Prof Zhu, a self-professed optimist, every difficult moment in research is an opportunity to get a better feel of it.