Research

Breakthrough

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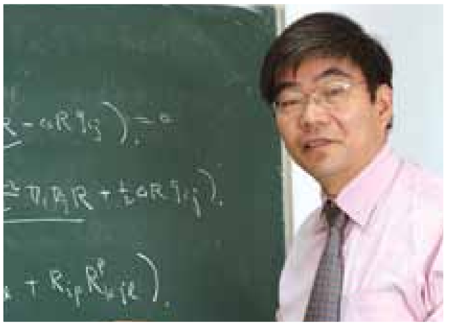

Scalar Curvature Flow Method Resolves Prescribed Scalar Curvature Problem

A long-standing problem in mathematics is how to continuously deform a rough sphere into a perfect sphere. Different methods have been proposed for measuring the roughness of a sphere.

The scalar curvature measurement has proven to be the weakest, resulting in the prescribing scalar curvature problem. This problem can be converted to finding positive solutions of elliptic-type partial differential equations. However, the bubbling phenomenon (a very sharp needle) can occur.

Prof Xu Xingwang and his team recently developed the scalar curvature flow method, based on the powerful Hamilton-Ricci flow method, to make the important breakthrough on this long standing problem.

The scalar curvature flow is based on three assumptions: first, the rough sphere is relatively flat; second, the rough sphere has no degenerate sharp needle; third, the scalar curvature of the rough sphere satisfies some topological identity.

The team's important observation is that the relative flatness proposed in the first assumption ensures that the flow has only one sharp needle, which makes the blow-up analysis easier.

The other tool Prof Xu uses in his work is the infinitely dimensional Morse theory. The difficulty in using this theory lies in controlling the energy level, as the global existence of the flow depends on the initial energy level.

The team found that it is possible to deform the open set with the controllable energy level in the infinite dimensional space into a set with one base point attached with several cells if for all initial data in the open set, the flow does not converge. The latter set has a simple topology. All that remains is a simple topological argument to get a contradiction with their third assumption.

Publication:

Chen Xuezhang and Xu Xingwang, The scalar curvature flow on Sn—perturbation theorem revisited. *Inventiones Mathematicae* 187(2), 395-506 (2012).

$$u_t = \frac{n-2}{4}(\alpha(t) f - R)u$$